Corrigendum pour “Partition Into Triangles on Bounded Degree Graphs”

Laurent Lyaudet*

7 juin 2025

Résumé

Dans cette note, nous pointons une erreur logique dans van Rooij et al. (2013) et nous montrons comment la corriger.

Version initiale : 2025/06/07 Version courante : 2025/06/07

Nous avons trouvé une erreur dans une preuve de van Rooij et al. (2013) et nous avons trouvé comment la corriger. Nous avons prévenus les auteurs (février 2022) qui ont dit qu'ils publieraient un corrigendum mais qui ne l'ont pas fait et n'ont pas répondu quand je les ai relancés en mai 2023. L'article initial est d'abord paru dans une conférence : van Rooij et al. (2011). Un article antérieur a une intersection non-vide sur les résultats : Caprara and Rizzi (2002).

Le déroulé des échanges de courriels n'est pas gênant et explique tout. Le voici :

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2022/02/05 22:25
Subject: Corrigendum to
         "Partition Into Triangles on Bounded Degree Graphs"
From: Laurent Lyaudet <laurent.lyaudet@gmail.com>
To: h.l.bodlaender@uu.nl, hansb@cs.uu.nl, jmmrooij@cs.uu.nl,
    markonie@xs4all.nl

Hello,

Thanks for writing this article
and making it available freely to anyone.
I have not finished reading it,
I am currently at the end of section 4.
But I wanted to tell you that Lemma 5 is false.
That's the bad news.
But the good news is that the statement of Lemma 5 can be modified
and the proof adapted so that the other results still stand.
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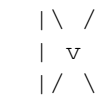
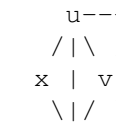
In detail, it is easy to see that local neighborhood 3b
can be combined with local neighborhood 2b.
In line 7 of the proof, this sentence is false :
"Thus, the top two vertices have a common neighbour in N[v]."



can easily be extended with a vertex x on the left :



If you forget the three vertices u, v, a that have already degree 4
(and 3b local neighborhood),
you have a graph CE with 3 vertices x, w, b of degree 2
that can be merged
with other vertices of degree 2 in other "building blocks".
Thus the simplest example of a connected component
mixing 2b and 3b local neighborhoods is made of two copies
of the graph CE above:
- u, w, x, v, a, b
- u', w', x', v', a', b'

where you identify x = x', w = w', b = b', thus these 3 vertices have
2b local neighborhood.

However, it is easy to see that whenever we have
an induced subgraph of G isomorphic to CE,
the only possibility for vertex v is to be in triangle uva,
otherwise u or a would be without triangle.

Thus Lemma 5 can be modified to Lemma 5' :
Let G be a 4-regular instance of Partition into triangles in which
the local neighborhood of each vertex equals
Case 2b, 3a, or 3b in Fig. 3.
Then :
- either there is no vertex with local neighborhood 3b,
- or some vertex with local neighborhood 3b has a neighbor
  with local neighborhood 2b,

and we can reduce G to a smaller equivalent instance in linear time
- or vertices whose local neighborhood 3b form separate connected
  components in G ;
  and in linear time, we can decide that G is a No-instance,
  or remove these components to obtain an equivalent smaller instance.

My goal with this email is that this corrigendum can be made
freely available on the internet.
Let me know if you do not want to publish it under your names
in Theory of Computing Systems or elsewhere.
In that case, I'll make it available on my webpage
and maybe on arXiv if they do not reject it.

Best regards,
     Laurent Lyaudet

2022/02/08 13:04
From: Marcel van Kooten Niekerk <markonie@xs4all.nl>
To: Laurent Lyaudet <laurent.lyaudet@gmail.com>, h.l.bodlaender@uu.nl,
    hansb@cs.uu.nl, jmmrooij@cs.uu.nl

Hello Laurent,

Thanks for your interest in our article.
I have read your interesting comments and will discuss it with the
other authors before sending a more in-depth answer.

Greetings,
Marcel van Kooten Niekerk

2022/02/08 23:18
From: Laurent Lyaudet <laurent.lyaudet@gmail.com>
To: Marcel van Kooten Niekerk <markonie@xs4all.nl>
Cc: h.l.bodlaender@uu.nl, hansb@cs.uu.nl, jmmrooij@cs.uu.nl

Hello Marcel,

Thanks for your response.
Since my previous email, I finished reading the core of the article
but did not read the appendix yet.
I enjoyed reading your article.
When doing some bibliographic search on packing cycles and triangles,
I read the following article before yours :
https://www.sciencedirect.com/science/article/abs/pii/S0020019002002740
You may find it interesting,
and there is a small overlap with your results.
But it does not diminish the interest of your article
with its nice reduction to Exact 3-SAT.

Best regards,
     Laurent Lyaudet

2022/02/25 23:41
From: Laurent Lyaudet <laurent.lyaudet@gmail.com>
To: Marcel van Kooten Niekerk <markonie@xs4all.nl>
Cc: h.l.bodlaender@uu.nl, hansb@cs.uu.nl, jmmrooij@cs.uu.nl

Hello,

Did you find the time to discuss my comments together ?

Best regards,
     Laurent Lyaudet

2022/03/22 11:21
From: Marcel van Kooten Niekerk <markonie@xs4all.nl>
To: Laurent Lyaudet <laurent.lyaudet@gmail.com>
Cc: h.l.bodlaender@uu.nl, hansb@cs.uu.nl, jmmrooij@cs.uu.nl

Hello Laurent,

We have discussed your comments.
We agree about the error you spotted in the article,
and we are going to publish a correction.
However, it is very busy at the moment with educational activities,
so we plan to work on it in the summer.

Greetings,
Marcel van Kooten Niekerk

2022/03/22 22:50
From: Laurent Lyaudet <laurent.lyaudet@gmail.com>
To: Marcel van Kooten Niekerk <markonie@xs4all.nl>
Cc: h.l.bodlaender@uu.nl, hansb@cs.uu.nl, jmmrooij@cs.uu.nl

Hello Marcel,

I’m glad I could help correct this error :)

Best regards,
     Laurent Lyaudet

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2023/05/09 23:31
From: Laurent Lyaudet <laurent.lyaudet@gmail.com>
To: Marcel van Kooten Niekerk <markonie@xs4all.nl>
Cc: h.l.bodlaender@uu.nl, hansb@cs.uu.nl, jmmrooij@cs.uu.nl

Hello Marcel,

I think you did not have time to publish a corrigendum last summer,
or I cannot find it on the Internet.
Any plan to do it this summer, or do you mind if I publish a
corrigendum on my website or arxiv ?

Best regards,
    Laurent Lyaudet

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En l'absence de réponse après ce dernier courriel, plus de deux ans après, je pense que c'est ok de publier ce corrigendum. Ils avaient juste des choses plus importantes à faire.

Merci Dieu ! Merci Père ! Merci Jésus ! Merci Saint-Esprit !

Références

A. Caprara and R. Rizzi. Packing triangles in bounded degree graphs. *Inf. Process. Lett.*, 84(4) :175–180, 2002. doi : 10.1016/S0020-0190(02)00274-0. URL https://doi.org/10.1016/S0020-0190(02)00274-0.

J. M. M. van Rooij, M. E. van Kooten Niekerk, and H. L. Bodlaender. Partition into triangles on bounded degree graphs. In I. Cerná, T. Gyimóthy, J. Hromkovic, K. G. Jeffery, R. Královic, M. Vukolic, and S. Wolf, editors, *SOFSEM 2011 : Theory and Practice of Computer Science - 37th Conference on Current Trends in Theory and Practice of Computer Science, Nový Smokovec, Slovakia, January 22-28, 2011. Proceedings*, volume 6543 of *Lecture Notes in Computer Science*, pages 558–569. Springer, 2011. doi : 10.1007/978-3-642-18381-2_46. URL https://doi.org/10.1007/978-3-642-18381-2_46.

J. M. M. van Rooij, M. E. van Kooten Niekerk, and H. L. Bodlaender. Partition into triangles on bounded degree graphs. *Theory Comput. Syst.*, 52(4) :687–718, 2013. doi : 10.1007/S00224-012-9412-5. URL https://doi.org/10.1007/s00224-012-9412-5.